3

Model 1: Subject As Single Factor

3.1 The Model

In this section, I am considering reliability experiments where each subject is scored by a different group of r raters[1]. The experimenter controls which subject will participate in the experiment. However, the raters who will rate the selected subject are not necessarily under the experimenter's control. Moreover, I will assume that the rater produces m measurements (or replicate measurements) on each subject. These replicate measurements will eventually be averaged to increase the precision of the ratings.

Shrout and Fleiss (1979) referred to this model as Model 1, and suggested the notations $\mathsf{ICC}(1,1)$ and $\mathsf{ICC}(1,r)$. $\mathsf{ICC}(1,1)$ indicates that the Intraclass Correlation Coefficient (ICC) is based on a single rating per subject and per rater (the single rating could well be the average of the m replicate measurements associated with the rater). $\mathsf{ICC}(1,r)$ on the other hand indicates that for each subject, all measurements from all r raters are averaged. The ICC in this case, is based on a single measurement per subject aggregated from individual rater measurements.

[1] If the number of raters is 3 for example, then we will have $r = 3$. But r could take any integer value greater than or equal 2.

It is commonly assumed in this model that participating subjects are from a larger population of subjects you are interested in. If that is the case, the subject will be considered a Random Effect and model 1 is referred to a One-Factor Random Effect Model. Instead of being selected randomly from a larger population, if the subjects are selected in a subjective manner or for convenience, then the subject is considered a Fixed Effect and model 1 is referred to a One-Factor Fixed Effect Model.

3.2 ICC for Model 1

The one-way Analysis of Variance (ANOVA) using the subject as single factor is used to obtain the ingredients needed to compute the Intraclass Correlation Coefficient. Actually, you do not even need to know what the ANOVA analysis is all about to be able to use Excel for obtaining these ingredients. I will show you in the next section, all the steps needed to get the job done. All you need to know right now is that the ANOVA analysis done with Excel will produce two numbers called the Target (or Subject) Mean of Squares (denoted by TMS), and the Error Mean of Squares (denoted by EMS).

Selection of Subjects from a Larger Subject Population

- **The Unit of Analysis is the Individual Rating**

  If the subjects are selected from a larger population of subjects, and if the individual score assigned by a rater to a subject is considered as the unit of analysis, then the ICC is calculated as follows:

$$\mathsf{ICC}(1,1) = \frac{\mathsf{TMS} - \mathsf{EMS}}{\mathsf{TMS} + (rm-1)\mathsf{EMS}}$$

  If the number of replicates is $m = 1$, then the above expres-

sion becomes identical to that of Shrout & Fleiss (1979).

▶ **The Unit of Analysis is the Mean of all Scores assigned to a Subject**

If the subjects are selected from a larger subject population, and all ratings pertaining to a subject are averaged, then the ICC is calculated with the following expression:

$$\mathsf{ICC}(1,r) = \frac{\mathsf{TMS} - \mathsf{EMS}}{\mathsf{TMS}}.$$

Subjects Not Part of a Larger Subject Population

▶ **The Unit of Analysis is the Individual Score**

If the subjects are not selected from any larger subject population of interest, then participating subjects are the only ones of interest. The ICC is calculated with the following expression:

$$\mathsf{ICC}(1,1) = \frac{\mathsf{TMS} - \mathsf{EMS}}{\mathsf{TMS} + \left(\dfrac{rmn}{n-1} - 1\right)\mathsf{EMS}}$$

where n = Number of subjects that participated in the experiment.

▶ **The Unit of Analysis is the Mean of all Scores assigned to a Subject**

If the subjects are not selected from a larger subject population, and all ratings pertaining to a subject are averaged, then the ICC is calculated with the following expression:

$$\mathsf{ICC}(1,r) = \frac{\mathsf{TMS} - \mathsf{EMS}}{\mathsf{TMS} + \mathsf{EMS}/(n-1)}$$

3.3 Computing ICC with MS Excel

To illustrate the calculation of the ICC using Excel, I chose to use the small dataset of ratings presented in Table 26.2 (page 591) of Portney & Watkins (2008), and shown in Figure 3.1. This dataset contains the ratings of 4 raters who assigned a numeric score from 1 to 10 to each of the 6 subjects.

Because I am applying Model 1 of Shrout & Fleiss (1979), consider that the subject were not necessarily scored by the same group of 4 raters. You may well be in a situation where of the 4 raters who scored subject 1, only 1 also scored subject 2, the remaining 3 raters having scored their first subject. It is also possible that 24 raters produced those ratings, each scoring a single subject.

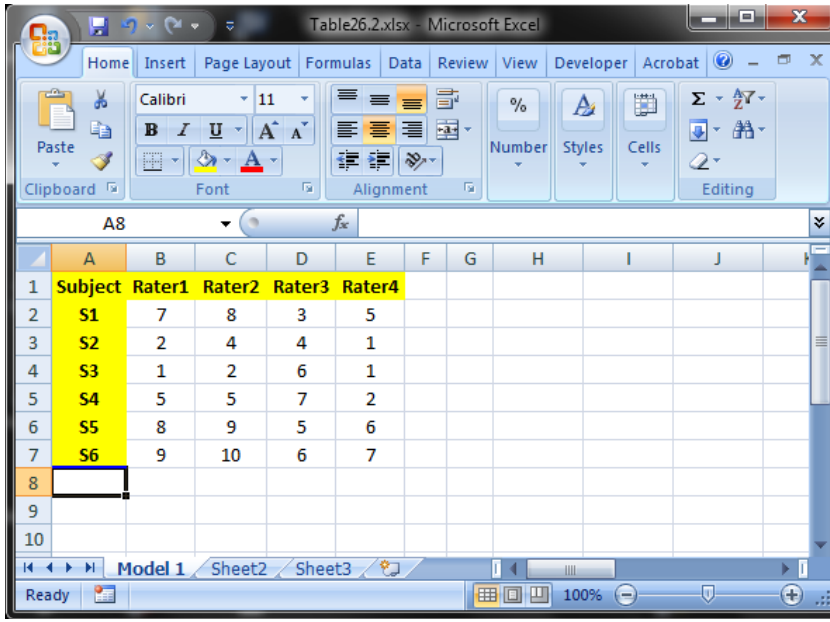

Figure 3.1. Rating Data from 4 Raters Across 6 Subjects

Under Model 1, the only thing we known with certainty is that 4 scores are available for each subject. Therefore, the subject varia-

tion can be investigated, and the **Subject** is the only factor affecting the score that we will be investigating. Identifying the factor being studied at this stage is essential for using Excel properly.

At this stage, I do not really need to know that the 6 subjects are from a larger subject population or not, nor what the unit of analysis will be. After all the different means of squares used to compute one ICC are the same needed to compute the other ICCs. I will show you how to obtain TMS and EMS. The only thing we know right now is that the raters have produced a single score for each rater as seen in Figure 3.1. That is the number of replicates is $m = 1$ (i.e. there is no replication).

1 **Launch the Analysis ToolPak** (see Figure 3.2)
Select the "Data" Tab from Excel's Menu Bar, then **Click on the "Data Analysis" Icon** from the Analysis Group. This will open up the list of Analysis Tools as shown in Figure 3.2.

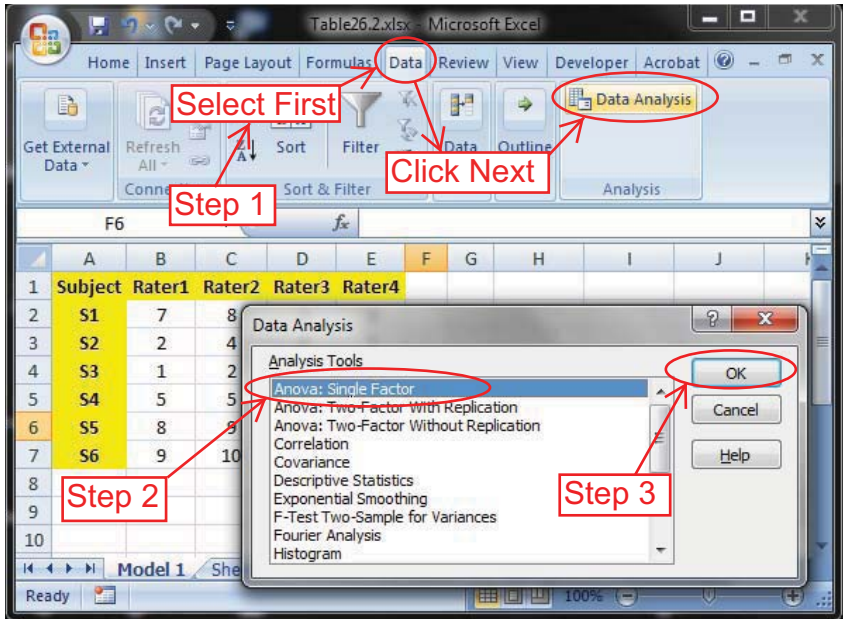

Figure 3.2. Selection of the ANOVA: Single Factor Module